COURSE DESCRIPTION CARD - SYLLABUS

Course name
Linear algebra with analitic geometry II

Course

Field of study
Mathematics in Technology

Year/Semester
1/2

Area of study (specialization)

Profile of study
general academic

Level of study
First-cycle studies

Course offered in
polish

Form of study
full-time

Requirements
compulsory

Number of hours

Lecture
30

Laboratory classes

Other (e.g. online)

Tutorials
30

Projects/seminars

Number of credit points

5

Lecturers

Responsible for the course/lecturer:
prof. dr hab. inż. Paweł Kolwicz;
pawel.kolwicz@put.poznan.pl

Responsible for the course/lecturer:

Prerequisites

Knowledge of linear algebra and analytical geometry from the first semester of this subject. Ability to apply matrix calculus, solve systems of linear equations, use vector calculus in analytical geometry, apply the basic notions of the theory of linear spaces and linear operators. He is aware of the need to expand his competences and is ready to cooperate

Course objective

Acquiring skills to analyze the determinateness of square form (bilinear form). Getting to know the basic notios of the space with an inner product (in particular the concept of an orthogonal basis). Second degree curves and surface recognition. Getting to know selected elements of differential geometry of curves.

Course-related learning outcomes

Knowledge

1. has knowledge of a bilinear (square) form, matrix of the form, scalar product, orthogonal basis, degree 2 curve (circle, parabola, hyperbola, ellipse), degree 2 surface (cylinder, cone, paraboloid, hyperboloid), the notion of a strictly tangent circle.

2. has knowledge about more important proofs, selected statements or the idea of proofs from the above area.

Skills

1. has the ability to find a matrix of quadratic (bilinear) forms in selected bases, study the determinateness of quadratic forms, calculate the inner product and the norm in the Euclidean space.

2. can use theorems to recognize curves and surfaces of the second degree, find the equation of a circle strictly tangent to a curve, determine the envelope of a family of curves for easy examples.

Social competences

1. can think and behave in good mathematical manner in the area of linear algebra and analitical geometry.

2. knows the limitation of own knowledge and understand the need of more far education and the necessity of systematic work.

Methods for verifying learning outcomes and assessment criteria

Learning outcomes presented above are verified as follows:

Lecture

-assessment of knowledge and skills on a written exam that checks the knowledge of notions and the ability to prove theorems and illustrate theories with examples (also possible short practical tasks).

Passing threshold: at least 50% of points. The issues for the exam, on the basis of which the questions are prepared, will be sent to students with the use of university electronic systems.

Tutorials:

- continuous assessment - rewarding activity (additional points) manifested in the discussion and cooperation in solving practical tasks,

- continuous assessment - rewarding the increase in the ability to use the techniques learned,

- obtaining additional points for activity during classes, including for the preparing presentations (discussing additional aspects of the issues, in particular the application of the discussed theory in other sciences or a reference to the location in the history of mathematics) and for comments on improving teaching materials

- active participation in consultations deepening knowledge and directing further work.

The knowledge acquired during the exercises is verified by two tests carried out on approx. 7 and 15 weeks (alternatively 1 test at the end of the semester). Passing threshold: at least 50% of points.

The rules for completing the course and the exact thresholds for passing the course will be provided to students at the beginning of the semester with the use of university electronic systems.

Programme content

Update: 30.08.2022r.

I. Lecture: theoretical issues (definitions, lemmas, theorems, conclusions, algorithms) and suitable examples for the following issues:

1.definition of a bilinear form, matrix of a bilinear form in the basis, rank of a bilinear form, symmetric and antisymmetric bilinear forms, definition of a square form, theorem about the relationship between a quadratic form and a corresponding symmetric bilinear form, matrix of a square form, canonical (diagonal) form of a square form, Lagrange's method of reducing the quadratic form to the canonical form, the real quadratic form in the normal form, determinateness of the quadratic form, Jacobi's method of reducing to the canonical form, Sylvester's criterion.

2.definition of an inner product, definition of Euclidean space, norm, Schwarz inequality, angle between vectors of Euclidean space, orthogonal vectors, triangle inequality, orthogonal basis (orthogonal basis), orthogonality and linear independence of vectors, Gram-Schmidt theorem on orthogonalization, theorem on decomposition of the Euclidean space, identity of the parallelogram.

3.algebraic and transcendental curves, parametric and polar form of a curve, polar coordinates, circle, ellipse, hyperbola, parabola - definitions, equations, parameters, directrices of a hyperbola (ellipse), theorem about eccentricity for ellipse and hyperbola, line of degree 2 - definition, examples, homogeneous equation st 2 - definition and theorem, basic theorem about degree 2 lines.

4. second degree surfaces, surfaces of revolution, equations: circular ellipsoid , hyperboloid of one (two) sheet(s), elliptical paraboloid, elliptical cone, definition of an elliptical, hyperbolic and parabolic cylinder and hyperbolic paraboloid, division of spatial objects of the second degree.

5.envelope of a family of curves, tangency of curves, strictly tangent circle, curvature of a plane curve, formulas for the center and radius of curvature, evolute and evelovent for a curve, curve in the 3-dimensional space, tangent line and plane normal to the curve, plane strictly tangent to the curve, rectifying plane , moving trihedral (Frenet's) - equations of three planes and three lines, curvature and torsion of a spatial curve.

II. Tutorials: solving practical problems illustrating the concepts discussed and example problems with the use of theoretical apparatus from the lecture, e.g .:

1. determining the matrix of the bilinear form in the basis, reducing the quadratic form to the canonical (diagonal) form using the Lagrange or Jacobi method.

2. checking the orthogonality of vectors, performing orthogonalization using the Gram-Schmidt method.

3. checking which line is represented by the second degree equation by the method of reducing to full squares or using the theorem, sketches of the graph of conic curves, taking into account important elements.

4. checking which second degree spatial object is represented by the equation by the method of reducing to full squares or by using the theorem, sketches of the graphs of the second degree surface by the plane cross-sectional method.

5. finding the envelopes, strictly tangent circle, individual elements of the moving trihedral.

Teaching methods

I. Lectures

1. a lecture conducted on the blackboard in an interactive way with the formulation of questions to a group of students, the lecture supplemented by a computer presentation

2.the activity of students is taken into account (preparation of historical talks on mathematicians related to the presented material, papers on the use of algebra in engineering sciences, presenting evidence left to be done on their own) during classes when issuing the final grade,

3.initiating discussions during the lecture,

4. theory presented in connection with the current knowledge of students from previous lectures.

II. Tutorials

1. solving example tasks on the blackboard

2. detailed reviewing of the solutions to the tasks by the tutor and discussion of the comments.

Bibliography

Basic

1. A. I. Kostrykin, Wstęp do algebry, cz.1 Podstawy algebry, PWN, Warszawa 2004.

2. A. I. Kostrykin, Wstęp do algebry, cz.2 Algebra liniowa, PWN, Warszawa 2004.

3. A. I. Kostrykin, Zbiór zadań z algebry, PWN, Warszawa 2005.

4. T. Jurlewicz, Z. Skoczylas, Algebra liniowa 1, Wrocław 2003.

5. T. Jurlewicz, Z. Skoczylas, Algebra liniowa 2, Wrocław 2005.

6. F. Leja, Geometria analityczna, PWN, Warszawa 1961.

Additional

1. H. Arodź, K. Rościszewski, Zbiór zadań z algebry i geometrii analitycznej dla fizyków, PWN, 1990.

Breakdown of average student's workload

	Hours	ECTS
Total workload	125	5,0
Classes requiring direct contact with the teacher	63	2,5
Student's own work (literature studies, preparation for laboratory classes/tutorials, preparation for tests/exam, project preparation) [1]	62	2,5

[1] delete or add other activities as appropriate